1S_FicheApplicationsDerivation2016

April 24, 2016

0.1 Fiche d'exercices Application de la dérivation 634 2015/2016

```
In [1]: from sympy import *
        init_session()
```

```
IPython console for SymPy 1.0 (Python 3.4.3-32-bit) (ground types: python)

These commands were executed:
>>> from __future__ import division
>>> from sympy import *
>>> x, y, z, t = symbols('x y z t')
>>> k, m, n = symbols('k m n', integer=True)
>>> f, g, h = symbols('f g h', cls=Function)
>>> init_printing()

Documentation can be found at http://docs.sympy.org/1.0/
```

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
```

```
In [3]: % matplotlib inline
```

```
In [46]: def deriver(exp, ordre = 1):
             return diff(exp, x, ordre)

         def simplifier(exp):
             return simplify(exp)
```

```
In [34]: def tracer(exp, xmin, xmax):
             f = lambdify(x, exp, "numpy")
             lesx = np.linspace(xmin, xmax, 101)
             lesy = f(lesx)
             plt.grid(True)
             plt.axhline(0, color='red')
             plt.axvline(0, color='red')
             plt.plot(lesx, lesy)
```

0.2 Exercice 1

```
In [6]: fx = x**3 - 3*x**2 - 7*x
```

```
In [7]: fx
```

```
Out[7]:
```

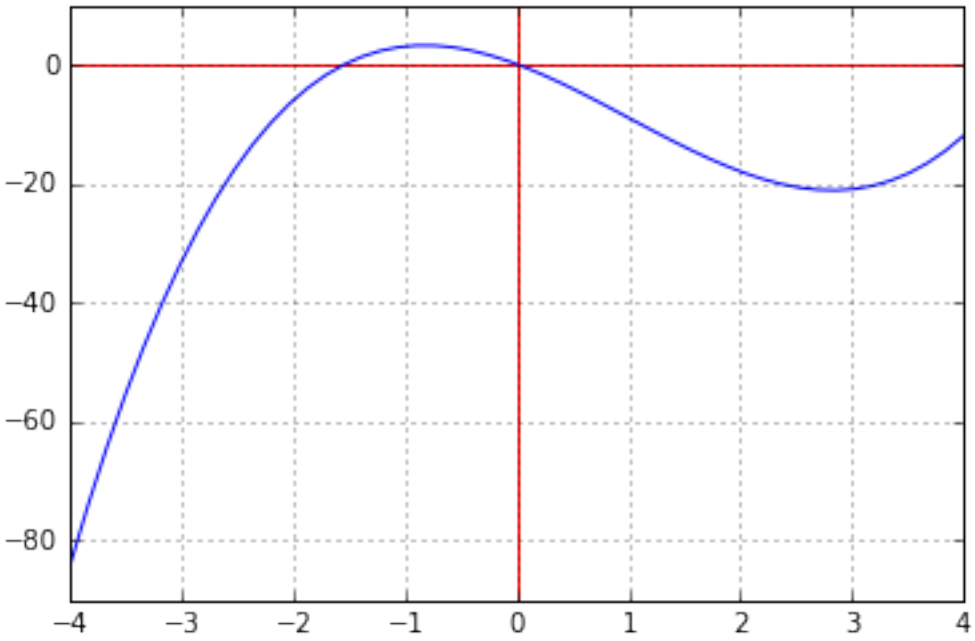
$$x^3 - 3x^2 - 7x$$

In [8]: deriver(fx)

Out[8]:

$$3x^2 - 6x - 7$$

In [16]: tracer(fx, -4, 4)



In [17]: gx = (x**2 + 1)*sqrt(x)

In [18]: gx

Out[18]:

$$\sqrt{x}\left(x^2 + 1\right)$$

In [19]: deriver(gx)

Out[19]:

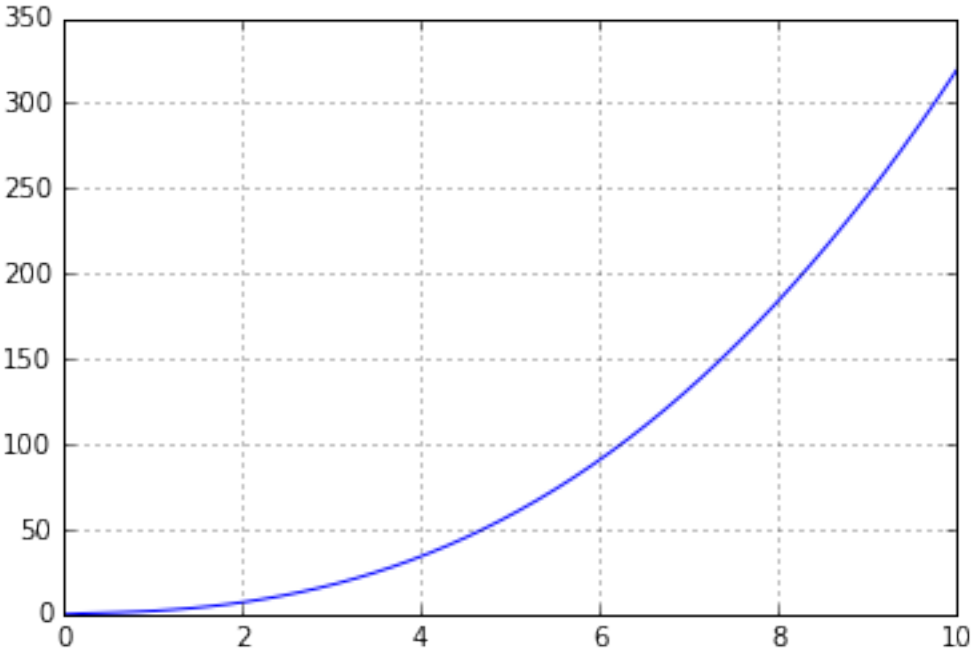
$$2x^{\frac{3}{2}} + \frac{x^2 + 1}{2\sqrt{x}}$$

In [28]: simplifier(deriver(gx))

Out[28]:

$$\frac{5x^2 + 1}{2\sqrt{x}}$$

In [29]: tracer(gx, 0, 10)



In [30]:
```
hx = (x + 4)/x**2
```

In [31]:
```
hx
```

Out[31]:

$$\frac{1}{x^2}\left(x+4\right)$$

In [32]:
```
deriver(hx)
```

Out[32]:

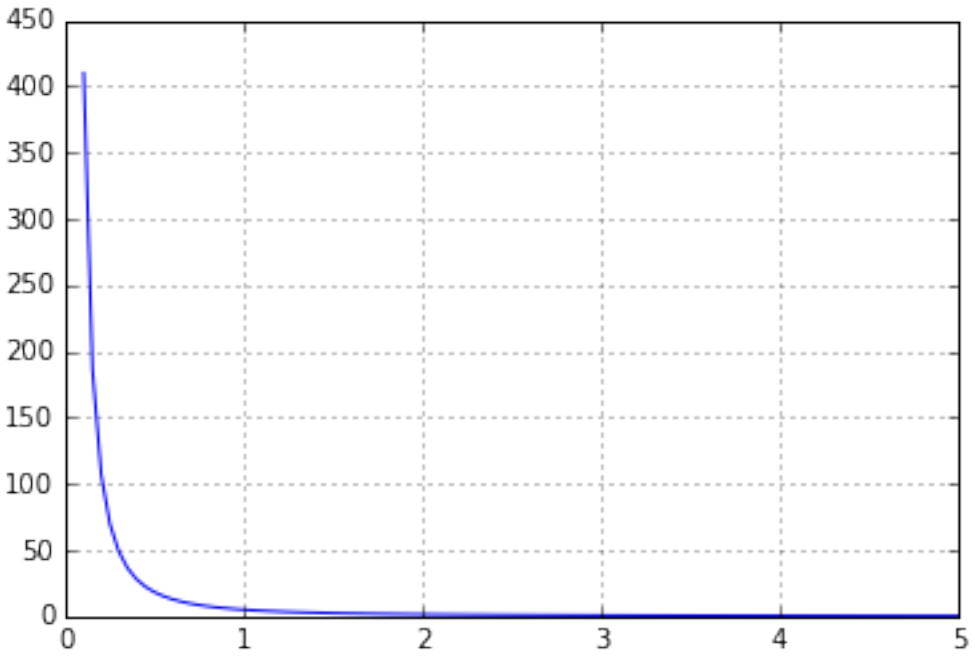
$$\frac{1}{x^2} - \frac{1}{x^3}\left(2x+8\right)$$

In [33]:
```
simplifier(hx)
```

Out[33]:

$$\frac{1}{x^2}\left(x+4\right)$$

In [35]:
```
tracer(hx, 0.1, 5)
```



In [36]:
```
jx = (3*x - 1)**4
```

In [37]:
```
jx
```

Out[37]:

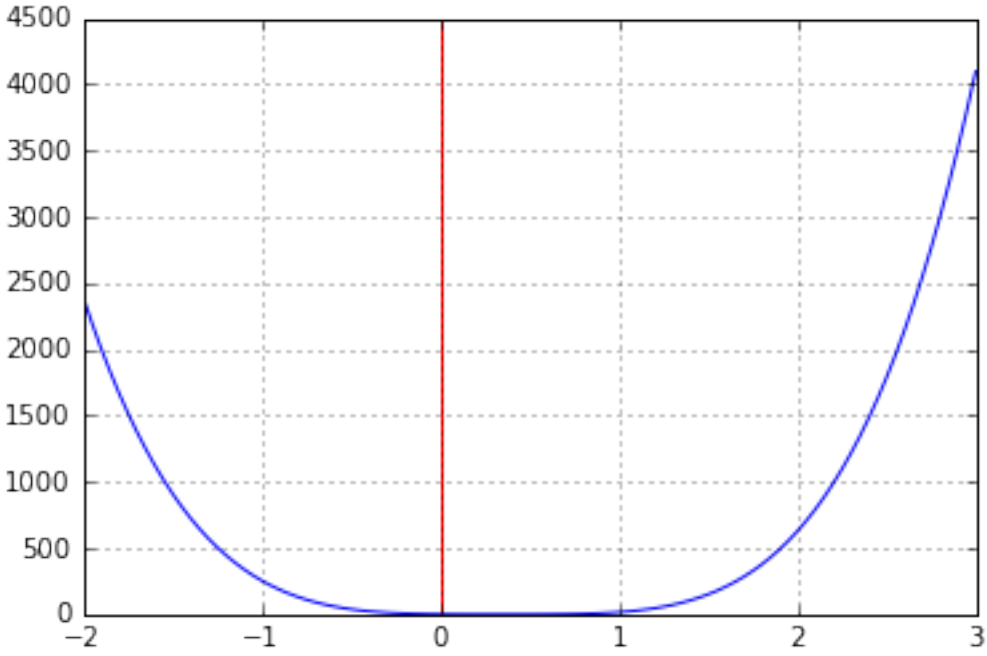
$$(3x - 1)^4$$

In [38]:
```
deriver(jx)
```

Out[38]:

$$12\,(3x - 1)^3$$

In [40]:
```
tracer(jx, -2, 3)
```



In [41]:
```
kx = sqrt(x)/(x + 6)
```

In [42]:
```
kx
```

Out[42]:

$$\frac{\sqrt{x}}{x+6}$$

In [43]:
```
deriver(kx)
```

Out[43]:

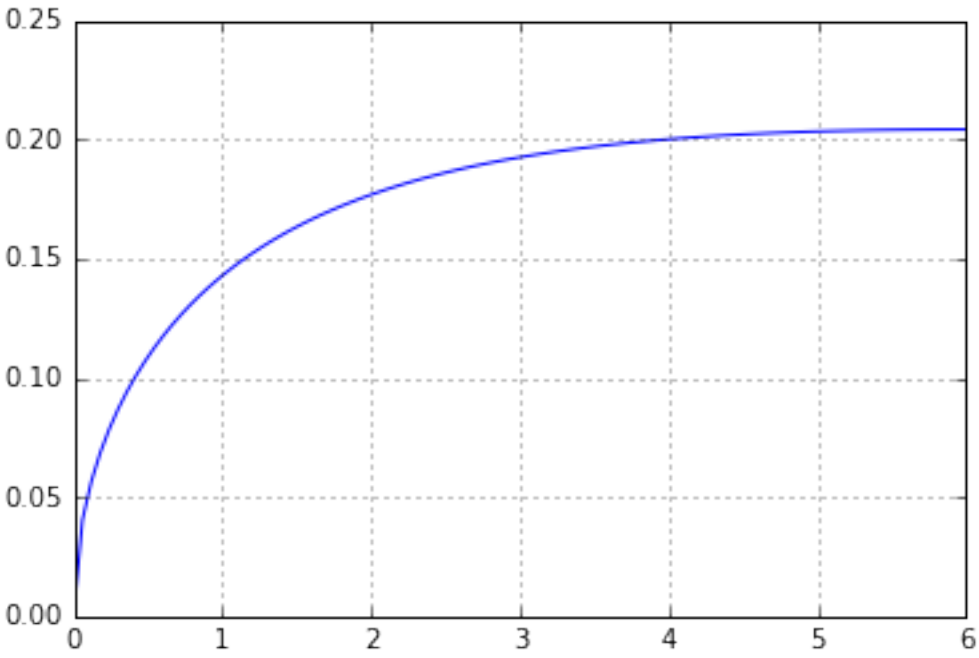
$$-\frac{\sqrt{x}}{(x+6)^2}+\frac{1}{2\sqrt{x}\,(x+6)}$$

In [44]:
```
simplifier(deriver(kx))
```

Out[44]:

$$\frac{-x+6}{2\sqrt{x}\,(x+6)^2}$$

In [45]:
```
tracer(kx, 0, 6)
```



In [47]:
```
mx = 3*x**4 + 8*x**3 - 78*x**2 + 120*x - 200
```

In [48]:
```
mx
```

Out[48]:

$$3x^4 + 8x^3 - 78x^2 + 120x - 200$$

In [50]:
```
d1mx = deriver(mx, 1)
d1mx
```

Out[50]:

$$12x^3 + 24x^2 - 156x + 120$$

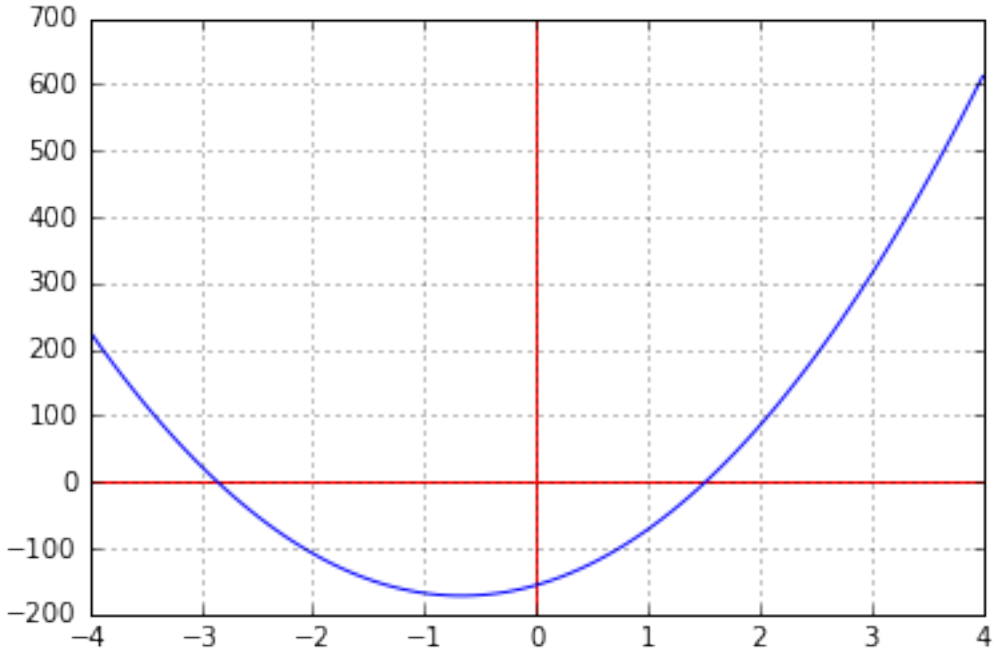
In [51]:
```
d2mx = deriver(mx, 2)
```

In [52]:
```
d2mx
```

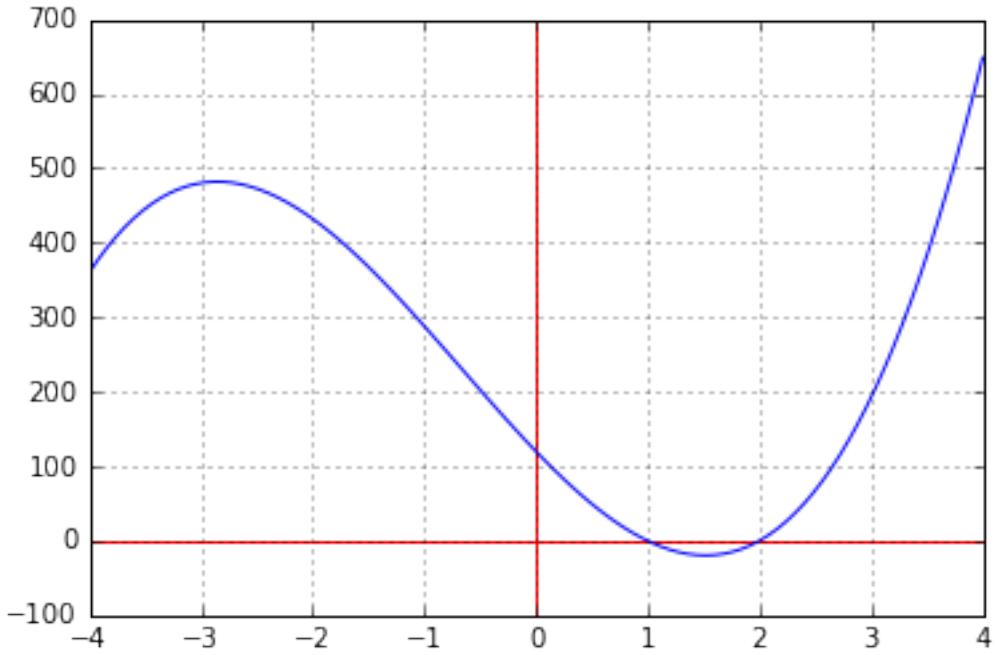
Out[52]:

$$12\left(3x^2 + 4x - 13\right)$$

In [53]:
```
tracer(d2mx, -4, 4)
```



In [55]: `tracer(d1mx, -4, 4)`

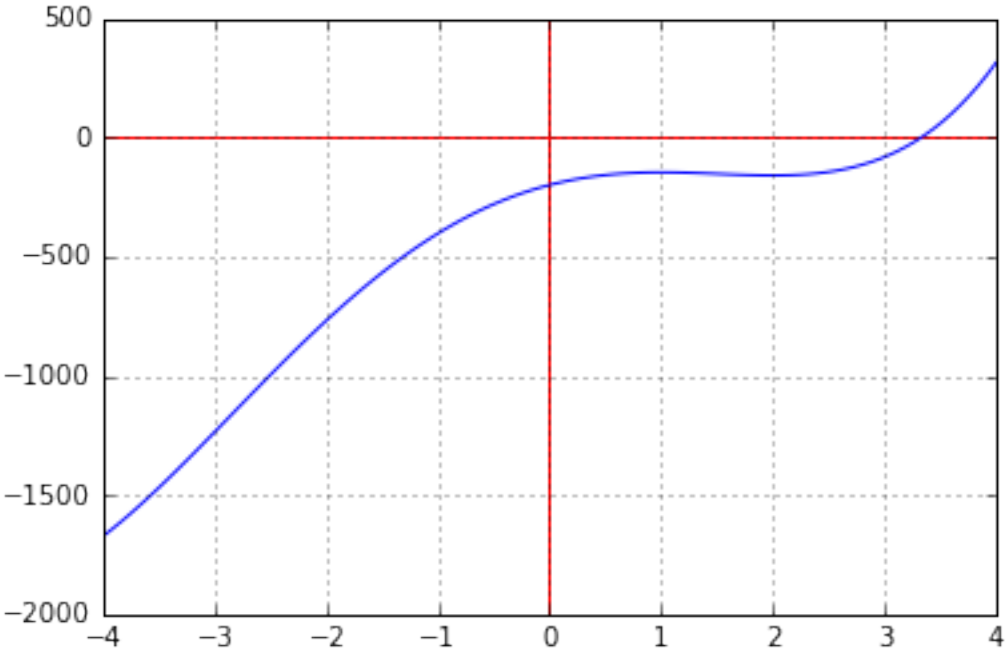


In [56]: `factor(d1mx)`

Out[56]:

$$12\,(x-2)\,(x-1)\,(x+5)$$

In [57]:
```
tracer(mx, -4, 4)
```



0.3 Exercice 2

In [58]:
```
fx = x + 3/x
```

In [59]:
```
deriver(fx)
```

Out[59]:

$$1 - \frac{3}{x^2}$$

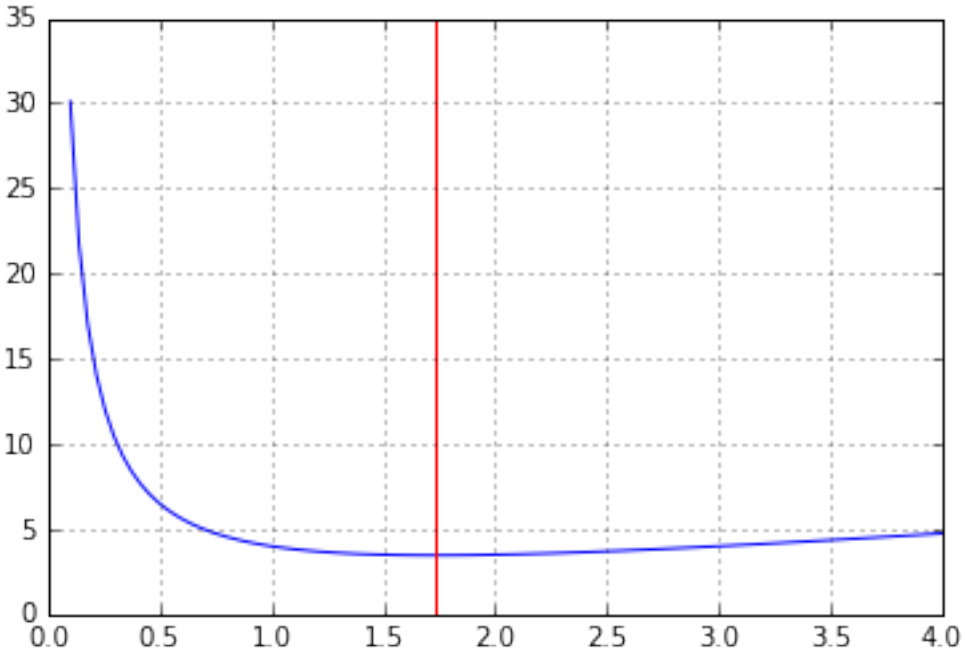
In [61]:
```
factor(deriver(fx))
```

Out[61]:

$$\frac{1}{x^2}\,(x^2-3)$$

In [63]:
```
tracer(fx, 0.1, 4)
plt.axvline(np.sqrt(3), color='red')
```

Out[63]:
```
<matplotlib.lines.Line2D at 0xa9c1abcc>
```



0.4 Exercice 3

In [64]:
```
fx = -Rational(2, 3)*x**3 -2*x**2 + 4*x - Rational(1, 3)
```

In [65]:
```
fx
```

Out[65]:

$$-\frac{2x^3}{3} - 2x^2 + 4x - \frac{1}{3}$$

In [66]:
```
deriver(fx)
```

Out[66]:

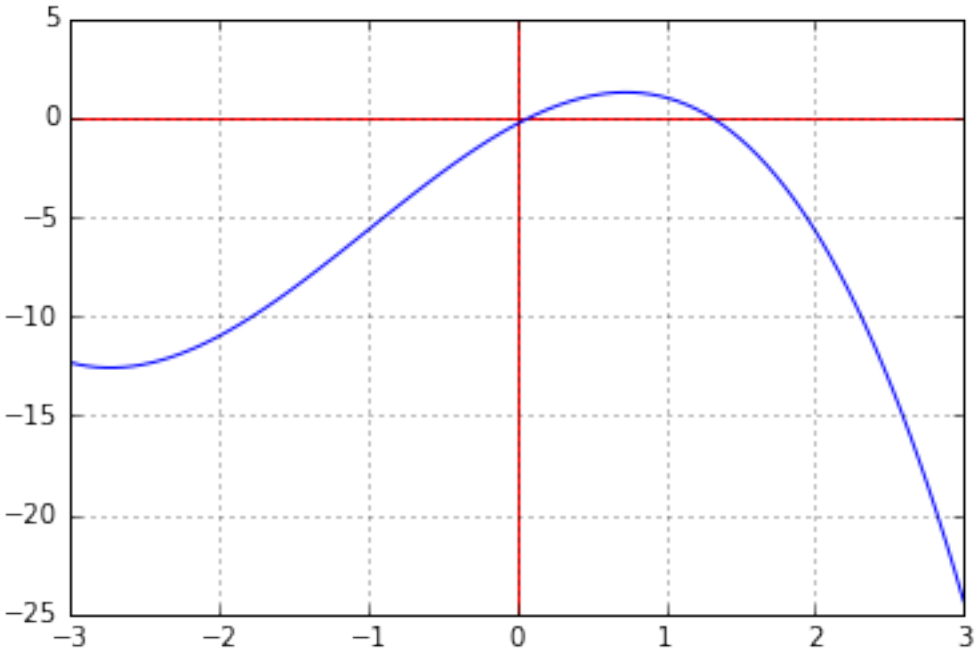
$$-2x^2 - 4x + 4$$

In [68]:
```
solve(deriver(fx), x)
```

Out[68]:

$$\left[-1 + \sqrt{3}, \quad -\sqrt{3} - 1\right]$$

In [70]:
```
tracer(fx, -3, 3)
```



0.5 Exercice 4

In [71]:
```
hx = (x**2 - 1)/(x**2 + 2)
```

In [72]:
```
hx
```

Out[72]:

$$\frac{x^2 - 1}{x^2 + 2}$$

In [73]:
```
deriver(hx)
```

Out[73]:

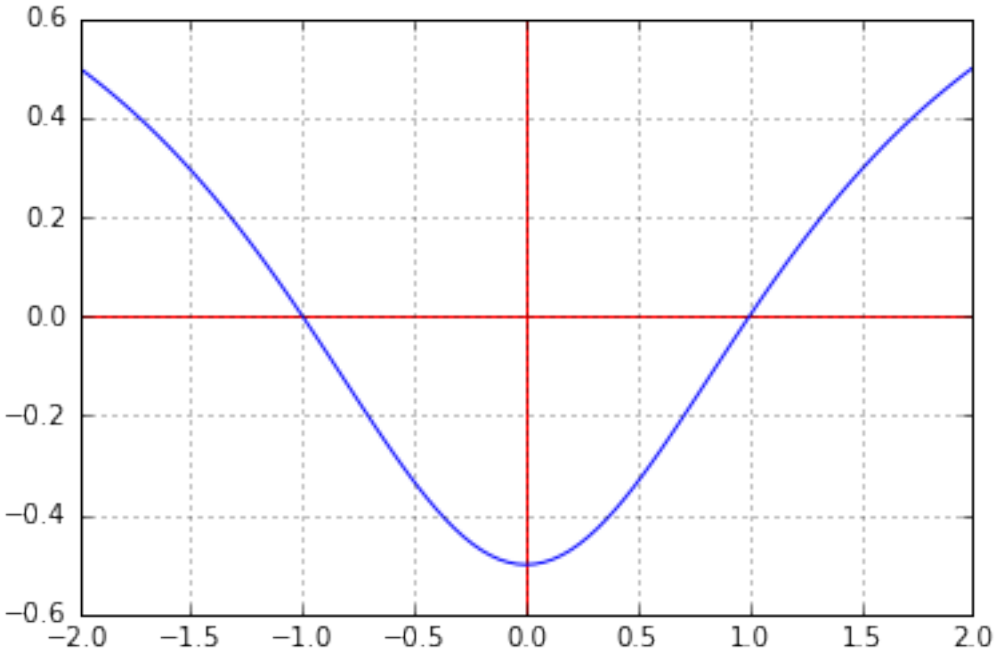
$$-\frac{2x\left(x^2 - 1\right)}{\left(x^2 + 2\right)^2} + \frac{2x}{x^2 + 2}$$

In [74]:
```
simplifier(deriver(hx))
```

Out[74]:

$$\frac{6x}{\left(x^2 + 2\right)^2}$$

In [75]:
```
tracer(hx, -2, 2)
```



0.6 Exercice 5

In [81]:
```
V = symbols('V')
```

In [82]:
```
ax = 2*(pi*x**2 + V/x)
```

In [83]:
```
ax
```

Out[83]:

$$\frac{2V}{x} + 2\pi x^2$$

In [84]:
```
deriver(ax)
```

Out[84]:

$$-\frac{2V}{x^2} + 4\pi x$$

In [85]:
```
factor(deriver(ax))
```

Out[85]:

$$\frac{1}{x^2}\left(-2V + 4\pi x^3\right)$$

In [88]:
```
s = solve(deriver(ax), x)
```

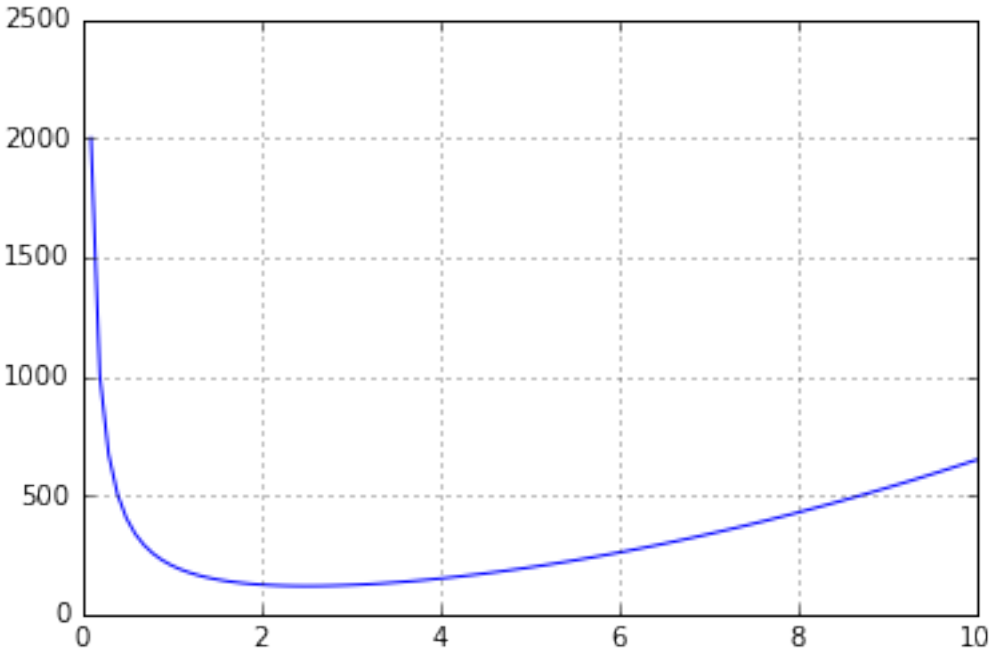
In [89]:
```
s[0] #valeur de x annulant la dérivée et rayon du volume minimal
```

Out[89]:

$$\frac{2^{\frac{2}{3}}\sqrt[3]{V}}{2\sqrt[3]{\pi}}$$

In [97]:
```
#Exemple pour V = 100
ax100 = ax.subs(V, 100)
```

In [98]:
```
tracer(ax100, 0.1, 10)
```



0.7 Exercice 8

In [99]:
```
fx = sqrt(x)/(x + 4)
```

In [100]:
```
fx
```

Out[100]:

$$\frac{\sqrt{x}}{x+4}$$

In [101]:
```
deriver(fx)
```

Out[101]:

$$-\frac{\sqrt{x}}{(x+4)^2} + \frac{1}{2\sqrt{x}\,(x+4)}$$

In [103]:
```
simplifier(deriver(fx))
```

Out[103]:

$$\frac{-x+4}{2\sqrt{x}\,(x+4)^2}$$

In [104]: `tracer(fx, 0.1, 16)`